HEAT TRANSFER OF A CYLINDER IN A COMPLEX SOUND FIELD

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It is well known that the heat-transfer process in a sound field is intensified in comparison with the stationary case, in which case it was shown [1] that these changes are due to stationary secondary flows. The case of a simple sound field is investigated in the overwhelming majority of cases. As shown in [2], however, the structure of stationary secondary flows in a complex sound field is considerably modified, which must be reflected in both local and integral characteristics of the heat-transfer process.

Consider heat transfer of a circular cylinder of radius a, placed in a high-frequency complex sound field consisting of two plane waves. The surface temperature of the cylinder \widetilde{T}_W and of the surrounding medium \widetilde{T}_{∞} is assumed to be constant, while the temperature difference $(\widetilde{T}_W - \widetilde{T}_{\infty})$ is assumed to be so small that the change in physical properties of the fluid, as well as natural convection, can be neglected. Neglecting also dissipation effects, the energy equation is written in the form

$$\frac{\partial T}{\partial \tau} + \frac{\varepsilon}{1 + r} \frac{\partial (\psi, T)}{\partial (r, \theta)} = \frac{\varepsilon^2}{\Pr Re_{st}} \nabla^2 T$$
(1)

with boundary conditions

$$T = 1$$
 for $r = 0$, $T = 0$ for $r \to \infty$, (2)

where $\mathbf{T} = (\widetilde{\mathbf{T}} - \widetilde{\mathbf{T}}_{\infty})/(\widetilde{\mathbf{T}}_{w} - \widetilde{\mathbf{T}}_{\infty})$. The remaining quantities are defined in [2].

As $\varepsilon \ll 1$, by using a perturbation method we reach a solution of Eq. (1) in form of the series

$$T = T_0 - \varepsilon T_1 - O(\varepsilon^2)$$

Using a similar expansion for the stream function ψ and substituting in Eq. (1), we obtained [3]

$$\frac{1}{1+r} \frac{\partial \left(\psi_1^{(\text{st})}, T_0\right)}{\partial \left(r, \theta\right)} = \frac{1}{\Pr \operatorname{Re}_{\text{st}}} \nabla^2 T_0,$$
(3)

where $\psi_1^{(st)}$ is the stream function of stationary secondary flow in the external region.

The solution of Eq. (3) must satisfy the second condition of relation (2), and for $r \rightarrow 0$ is asymptotically matched with the solution in the internal region.

Using the internal variables defined by Eq. (7) of [2], as well as putting $\text{Re}_{\text{st}} = O(1)$ and $(\epsilon^2 Pr) = O(1)$, Eq. (1) is written in the form [3]

$$\frac{\partial F}{\partial \tau} + \varepsilon \left(1 - \frac{\varepsilon \sqrt{2}}{\operatorname{Re}_{\mathrm{st}}} \eta \right) \frac{\partial (m, F)}{\partial (\eta, \theta)} = \frac{\varepsilon^2}{2 \left(\varepsilon^2 \operatorname{Pr} \right)} \frac{\partial^2 F}{\partial \eta^2} + O \left(\varepsilon^3 \right), \tag{4}$$

where F is the temperature in the internal region (Stokes layer). We seek a solution of (4) in the form

$$F = F_0 + \varepsilon F_1 + O(\varepsilon^2).$$

It was shown [3] that the first expansion term is the solution of the equation

$$\frac{\partial \left(m_{10}^{(\text{st})}, F_{0}\right)}{\partial \left(\eta, \theta\right)} = \frac{1}{2\left(\varepsilon^{2} \mathrm{Pr}\right)} \frac{\partial^{2} F_{0}}{\partial \eta^{2}},$$
(5)

Kazan. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 91-99, September-October, 1979. Original article submitted October 17, 1978.

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UDC 536.25

where $m_{i0}^{(st)}$ is the stream function of the internal secondary flow.

It follows from Eq. (5) that if $(\epsilon^2 Pr) \ll 1$, the internal region is essentially thermally conducting. When $(\epsilon^2 Pr) \gg 1$, the width of the temperature layer is considerably less than the width of the Stokes layer, and internal flows play an important role in the heat-transfer process. This fact was first established in [1].

When the heat-transfer process is determined by external flows, the temperature field is described by Eq. (3). It can be seen that for $(PrRe_{st}) \gg 1$ the whole external thermal region has the nature of a boundary layer whose width is on the order of $O[a(PrRe_{st})^{-1/2}]$. Taking this fact into account, we also introduce variables corresponding to a thermal boundary layer, in which the temperature is O(1) and the stream function $O(\varkappa)$, where $\varkappa = (PrRe_{st})^{-1/2} \ll 1$. Then

$$t_0(Y, \theta) = T_0(r, \theta), \ Y = \varkappa^{-1} r, \ \psi_{10}^{(st)} = \varkappa \widetilde{\psi}_{10}^{(st)}.$$
 (6)

We note that since in what follows we use the solutions of the hydrodynamical part of the problem, obtained under the assumption $\text{Re}_{\text{st}} \ll 1$, the condition $(\text{PrRe}_{\text{st}}) \gg 1$ implies that we consider the case of large Prandtl numbers, i.e., $\text{Pr} \gg 1$.

The internal expansion of the external stream function $\psi_{10}^{(st)}$ is written in the form

$$\widetilde{\psi}_{10}^{(\mathbf{g}_1)} = z_1 + \varkappa z_2 + O(\varkappa^2), \tag{7}$$

where

$$z_1 = Y\left(rac{\partial \widetilde{\psi}_{10}^{(st)}}{\partial Y}
ight)_{Y=0}; \ z_2 = rac{1}{2} Y^2 \left(rac{\partial^2 \widetilde{\psi}_{10}^{(st)}}{\partial Y^2}
ight)_{Y=0}.$$

Using the smallness condition of \varkappa , the solution of Eq. (3) is expanded in the series

$$t_0 = t_{00} + \varkappa t_{01} + O(\varkappa^2). \tag{8}$$

Substituting (7) and (8) into Eq. (3) and restricting ourselves to first-order terms in \varkappa , we obtain the following equation in terms of the variables of Eq. (6):

$$\frac{\partial \left(z_{1}, t_{00}\right)}{\partial \left(Y, \theta\right)} = \frac{\partial^{2} t_{00}}{\partial Y^{2}} \tag{9}$$

with boundary conditions

$$t_{00} = 0 \text{ for } Y \to \infty, t_{00} = 1 \text{ for } Y = 0.$$
 (10)

The second boundary condition of (10) follows from the fact the region of the Stokes layer is essentially thermally conducting, due to which one can neglect the temperature change in this region. This is satisfied by the condition $(\epsilon^2 Pr) \ll 1$, which imposes an upper bound on the Prandtl number.

Since the analytic representation of $\psi_{10}^{(st)}$ depends on the relation between the frequencies of the two waves [2], we consider the case of different frequencies. We place the coordinate system at the leading point of external secondary flows at the cylinder surface, introducing the variable

$$\sigma_1 = 2\theta - \arctan\left[\frac{B^2 b^{-2} \sin 2\theta_1}{1 + B^2 b^{-2} \cos 2\theta_1}\right] + \pi.$$

Using (7), as well as relation (25) of [2], Eq. (9) is written in the form

$$3N_1 \sin \sigma_1 \frac{\partial t_{00}}{\partial \sigma_1} - 3N_1 Y \cos \sigma_1 \frac{\partial t_{00}}{\partial Y} = \frac{\partial^2 t_{00}}{\partial Y^2},$$

whose solution is

$$t_{00} = 1 - \frac{2}{\Gamma(1/2)} \int_{0}^{x_{1}} e^{-\xi^{2}} d\xi, \qquad (11)$$

where

$$x_1 = \left(\frac{3N_1}{2}\right)^{1/2} Y \cos\left(\sigma_1/2\right); \ N_1 = \left(1 + 2B^2b^{-2}\cos 2\theta_1 + B^4b^{-4}\right)^{1/2};$$

and $\Gamma(\alpha)$ is the gamma function. Using (11), we calculate the local and integral heat-transfer coefficients, evaluated at the cylinder radius

$$Nu_{a} = \left(\frac{6}{\pi}\right)^{1/2} (\Pr \operatorname{Re}_{st})^{1/2} (1 + 2B^{2}b^{-2}\cos 2\theta_{1} + B^{4}b^{-4})^{1/4} |\cos(\sigma_{1}/2)|,$$

$$\overline{Nu}_{a} = 0.88 (\Pr \operatorname{Re}_{st})^{1/2} (1 + 2B^{2}b^{-2}\cos 2\theta_{1} + B^{4}b^{-4})^{1/4}.$$
(12)

It can be noted that for B = 0 expression (12) transforms to the dependence characterizing the heat-transfer process of a circular cylinder in a simple sound field [1]

$$Nu_{a} = \left(\frac{6}{\pi}\right)^{1/2} (\Pr \operatorname{Re}_{st})^{1/2} |\cos(\theta + \pi/2)|,$$

$$\overline{Nu}_{a} = 0.88 (\Pr \operatorname{Re}_{st})^{1/2} = 0.88 \frac{A_{1}}{\sqrt{\omega, \mathcal{D}}},$$
(13)

where \mathcal{D} is the thermal-conductivity coefficient of the fluid and A_1 and ω_1 are the velocity amplitude and frequency in the first wave. In expressions (12), (13) the absolute value is chosen, since the thermal conductivity coefficient is a positive quantity.

Thus, the analytic dependence describing heat transfer of a cylinder in a complex sound field for the case of different oscillation frequencies differs from the analog expressions for a simple sound field only by the presence of an additional factor of the form

$$(1 + 2B^2b^{-2}\cos 2\theta_1 + B^4b^{-4})^{1/4}$$

while the nature of the distribution of the local heat-transfer coefficient over the cylinder surface does not change, the distribution is symmetric with respect to the line passing through the extremum and the center of the cylinder (Fig. 1a), and the maximum of the heat-transfer coefficient coincides with the leading point of external flows at the cylinder surface.

Consider the conditions under which the presence of a second oscillatory motion leads to enhancement of the heat-transfer process in comparison with the case of a simple sound field, i.e.,

$$1 + 2B^{2}b^{-2}\cos 2\theta_{1} + B^{4}b^{-4} > 1 \text{ or } (Bb^{-1})^{2} > -2\cos 2\theta_{1}.$$
(14)

It follows from the latter relation, in particular, that independently of the amplitude-frequency relations the heat-transfer process in a complex sound field occurs more intensely than in a simple field if the angle between the propagation directions of the two waves satisfies the condition

$$|\theta_1| \leq \pi/4 + \pi n \ (n=0, 1, \ldots).$$



Fig.1

It can be verified that for $\theta_1 = 0$ or π there is maximum enhancement of heat transfer for assigned values of b and B. Besides, if $(Bb^{-1})^2 > 2$, the presence of a second oscillation also leads to enhancement of heat transfer independently of the value of θ_1 . When relation (14) is not satisfied the presence of a second oscillation worsens heat transfer in comparison with the case of a simple sound field, while this situation can be realized when the presence of a second oscillation completely suppresses convective heat transfer. This occurs when

$$1 + 2B^2 b^{-2} \cos 2\theta_1 + B^4 b^{-4} = 0 \tag{15}$$

or

$$B = b, \ \theta_1 = \pi (2n + 1)/2 \ (n = 0, 1, ...)$$

Thus, when the planes of both oscillations are perpendicular to each other and the amplitude-frequency relations obey Eq. (15), a cylinder placed in a complex sound field does not exchange energy with the surrounding space. This implies that heat-transfer processes by convection occur significantly more slowly than by thermal conductivity. At first glance this odd pattern is explained by the fact that under the conditions enumerated above, the external secondary flows generated by each oscillatory motion cancel each other, accurately up to terms of order $O(\delta_{ac}/a)$.

Consider the case of identical frequencies. Placing the coordinate system at the leading point of external secondary flows and using relation (32) of [2], as well as Eq. (7), Eq. (9) is written in the form

$$3N_{2}\left[\sin\left(\sigma_{2}\pm\beta\right)\mp A\right]\frac{\partial t_{00}}{\partial \sigma_{2}}-3N_{2}Y\cos\left(\sigma_{2}\pm\beta\right)\frac{\partial t_{00}}{\partial Y}=\frac{\partial^{2}t_{00}}{\partial Y^{2}},$$
(16)

where $\beta = \arcsin A$; $A = (2B \sin \theta_1 \cdot \sin \varphi)/N_2$, and

$$N_{2} = [(1 + B^{2} \cos 2\theta_{1} + 2B \cos \varphi \cdot \cos \theta_{1})^{2} + (B^{2} \sin 2\theta_{1} + 2B \sin \theta_{1} \cos \varphi)^{2}]^{1/2};$$

$$\sigma_{2} = \pm [2\theta - \beta - \arctan(D/C) - \pi]; D = B \sin \theta_{1} \cdot \cos \varphi$$

$$+ \frac{1}{2} B^{2} \sin 2\theta_{1}; C = \frac{1}{2} + B \cos \theta_{1} \cdot \cos \varphi + \frac{1}{2} B^{2} \cos 2\theta_{1},$$

where the plus sign refers to the case in which the fluid flow near the surface occurs clockwise, and the minus sign refers to counterclockwise flow.

Introducing the variable

$$x_2 = \left(\frac{3N_2}{4}\right)^{1/2} Y \frac{\sin\left(\sigma_2 \pm \beta\right) \mp A}{\left[\sqrt{1-A^2} - \cos\left(\sigma_2 \pm \beta\right) \mp A\sigma_2\right]^{1/2}},$$

Eq. (16) is reduced to an ordinary differential equation, whose solution is described by relation (11) with the only difference that x_1 must be replaced by x_2 . The expressions for the local and integral heat-transfer coefficients then acquire the form

$$Nu_{a} = \left(\frac{3}{\pi}\right)^{1/2} (PrRe_{st})^{1/2} N_{2}^{1/2} \frac{\sin(\sigma_{2} \pm \beta) \mp A}{\left[\sqrt{1 - A^{2}} - \cos(\sigma_{2} \pm \beta) \mp A\sigma_{2}\right]^{1/2}},$$

$$\overline{Nu}_{a} = \left(\frac{3}{\pi}\right)^{1/2} (PrRe_{st})^{1/2} N_{2}^{1/2} \left[(2\sqrt{1 - A^{2}} + 2A\beta - A\pi)^{1/2} + (2\sqrt{1 - A^{2}} + 2A\beta - A\pi)^{1/2} \right].$$
(17)

Figure 1 shows the distribution of the local heat-transfer coefficient over the surface of the cylinder for different values of the phase difference $(a - \varphi = 0, b - \varphi = 45^\circ)$, while

$$(\varepsilon^2 \operatorname{Pr}) \ll 1, \ \omega_1 = \omega_2, \ A_1 = A_2,$$

B = 1, $\theta_1 = 90^\circ$. Under certain conditions the distribution of the local heat-transfer coefficient is not symmetric with respect to the line passing through the leading point of the external secondary flows and the center of the cylinder. This asymmetry is due to the presence of large-scale circulatory motion, whose intensity is characterized by the parameter A. When A = 0 large-scale motion is absent and the nature of secondary flows, as well as the distribution of the local heat-transfer coefficient, coincide accurately up to corrective

factors with similar effects occurring in a simple sound field (see Fig. 1a). In particular, if B = 0, i.e., the second oscillation is absent, A = 0, $N_2 = 1$, and expressions (17) reduce to (13).

Excluding the case B = 0 from consideration, it can be said that the heat-transfer coefficient in a complex sound field is maximum for $\theta_1 = \varphi = 0$, while

$$\overline{\mathrm{Nu}}_{a} = 0.88(\mathrm{Pr} \ \mathrm{Re}_{\mathrm{st}})^{1/2}(1+B),$$

i.e., heat transfer is determined by the total velocity amplitude.

When B = 1, $\theta_1 = 0$, $\varphi = \pi$ (or $\theta_1 = \pi$, $\varphi = 0$) there is no oscillatory motion, since the oscillations cancel each other, and therefore there is no stationary motion of the fluid. This leads to the consequence $\overline{Nu}_{\alpha} = 0$.

Thus, the presence of large-scale circulatory flow leads to asymmetry in the distribution of the local heat-transfer coefficient, where to the extent of intensity enhancement of this motion (the parameter A increases) there is a suppression of Schlichting flow, and for A > 1 there is no region of reciprocal flow. We mention that relations (17) were obtained for $A \leq 1$. It can be shown that when A increases, heat exchange worsens, and for B = 1, $\theta_1 = \varphi = 90^\circ$ there is no heat exchange. Indeed, in this case Eq. (9) reduces to an equation of the thermal-conductivity type

$$-3\frac{\partial t_{00}}{\partial \theta} = \frac{\partial^2 t_{00}}{\partial Y^2}.$$

Since the boundary conditions (10) have no dependence on θ , the equation reduces to

$$d^2 t_{00}/dY^2 = 0,$$

whose solution, bounded at infinity, is trivial $(t_{00} = 0)$.

When the heat-transfer process is determined by internal secondary flows the temperature field is described by Eq. (5), while if $(\epsilon^2 Pr) \gg 1$, the thermal boundary layer is significantly smaller than the Stokes layer, and its transverse size is of the order of $O[a(\epsilon^2 Pr)^{-1/3}]$. Taking this into account, we introduce variables corresponding to the thermal boundary layer

$$h = \Delta^{-1}\eta, t_0(h, \theta) = F_0(\eta, \theta), \Delta = (\varepsilon^2 \operatorname{Pr})^{-1/3}.$$

We expand the current function m_{10}^{st} and the temperature t_0 in series:

$$n_{10}^{(\text{st})} = \Delta^2 \Omega_1 + \Delta^3 \Omega_2 + O(\Delta^4), \quad t_0 = t_{00} + \Delta t_{01} + O(\Delta^2), \quad (18)$$

where

$$\Omega_1 = \frac{1}{2} h^2 \left(\frac{\partial^2 m_{10}^{(\text{st})}}{\partial h^2} \right)_{h=0}; \ \Omega_2 = \frac{1}{6} h^3 \left(\frac{\partial^3 m_{10}^{(\text{st})}}{\partial h^3} \right)_{h=0}.$$

Substituting (18) into (5) and restricting the discussion to first-order terms in Δ , we obtain

$$\frac{\partial \left(\Omega_{1}, t_{00}\right)}{\partial \left(h, 0\right)} = \frac{1}{2} \frac{\partial^{2} t_{00}}{\partial h^{2}}$$
(19)

with boundary conditions

 $t_{00} = 0$ for $h \to \infty$, $t_{01} = 1$ for h = 0.

Consider the case of different frequencies. We place the coordinate system at the leading point of internal flows at the cylinder surface. Introducing then the variable

$$\sigma_{\mathbf{3}} = 2\theta - \operatorname{arctg}\left[\frac{B^2 b^{-1} \sin 2\theta_1}{1 + B^2 b^{-1} \cos 2\theta_1}\right],$$

as well as using expression (24) of [2], Eq. (19) is written in the form

$$2hN_3\sin\sigma_3\frac{\partial t_{00}}{\partial\sigma_3} - h^2N_3\cos\sigma_3\frac{\partial t_{00}}{\partial h} = \frac{1}{2}\frac{\partial^2 t_{00}}{\partial h^2},$$

whose solution is

$$t_{00} = 1 - \frac{3}{\Gamma(1/3)} \int_{0}^{x_{s}} e^{-\xi^{3}} d\xi_{s}$$
(20)

where

$$N_{3} = (1 + 2B^{2}b^{-1}\cos 2\theta_{1} + B^{4}b^{-2})^{1/2};$$
$$x_{3} = \left(\frac{4N_{3}}{\theta}\right)^{1/2}h \frac{\sin^{1/2}\sigma_{3}}{\left[\int\limits_{0}^{\sigma_{2}}\sin^{1/2}\chi d\chi\right]^{1/3}}.$$

The local and integral heat-exchange coefficients acquire then the form

$$Nu_{a} = 0.6 \left(\frac{A_{1}^{2}}{\sqrt{v\omega_{1}}} \frac{a}{\vartheta} \right)^{1/3} (1 + 2B^{2}b^{-1}\cos 2\theta_{1} + B^{4}b^{-2})^{1/6} \frac{\sin^{1/2}\sigma_{3}}{\left(\int\limits_{0}^{\sigma_{2}} \sin^{1/2}\chi d\chi\right)^{1/3}}$$

$$\overline{Nu}_{a} = 0.52 \left(\frac{A_{1}^{2}}{\sqrt{v\omega_{1}}} \frac{a}{\vartheta} \right)^{1/3} (1 + 2B^{2}b^{-1}\cos 2\theta_{1} + B^{4}b^{-2})^{1/6}.$$
(21)

It there is no secondary oscillatory motion (B = 0), expression (21) reduces to the relations describing the heat-exchange process in a simple sound field [1, 3]

$$\begin{split} \mathrm{Nu}_{a} &= 0.6 \left(\frac{A_{1}^{2}}{\sqrt{v\omega_{1}}} \frac{a}{\mathscr{D}} \right)^{1/3} \frac{\sin^{1/2} 2\theta}{\left(\int_{0}^{2\theta} \sin^{1/2} \chi d\chi \right)^{1/3}} \\ \overline{\mathrm{Nu}}_{a} &= 0.52 \left(\frac{A_{1}^{2}}{\sqrt{v\omega_{1}}} \frac{a}{\mathscr{D}} \right)^{1/3}. \end{split}$$
(22)

The coefficient obtained, however, in expression (22) for the integral coefficient of heat exchange differs from the result of [3]. For this reason we provide some detail.

Since the expression for the local heat-exchange coefficient (22) coincides with [1], the integral heatexchange coefficient is determined in the form

$$\overline{\mathrm{Nu}}_{a} = \frac{2}{\pi} \int_{0}^{\pi/2} \mathrm{Nu}_{a} d\theta = \frac{0.6}{\pi} \left(\frac{A_{1}}{\sqrt{\nu\omega_{1}}} \frac{a}{\varpi} \right)^{1/3} \int_{0}^{\pi} \frac{\sin^{1/2} k dk}{\left(\int \limits_{0}^{k} \sin^{1/2} \chi d\chi \right)^{1/3}},$$

where $k = 2\theta$. The integral is calculated as follows. We denote

$$z = \int_0^k \sin^{1/2} \chi d\chi, \ dz = \sin^{1/2} k dk;$$

then

$$\int_{0}^{\pi} \frac{\sin^{1/2} k dk}{\left(\int_{0}^{k} \sin^{1/2} \chi d\chi\right)^{1/3}} = \int_{0}^{\pi} \frac{dz}{z^{1/3}} = \frac{3}{2} z^{2/3} |_{0}^{\pi} = \frac{3}{2} \left(\int_{0}^{\pi} \sin^{1/2} \chi d\chi\right)^{2/3}.$$

Using [4]

$$\int_{0}^{\pi} \sin^{1/2} \chi d\chi = \frac{\pi^{1/2} \Gamma(3/4)}{\Gamma(5/4)} \simeq \frac{4}{3} \pi^{1/2},$$

we obtain the required result.

It follows from relationship (22) that the analytic dependences describing heat exchange of a cylinder in a complex sound field for the case of different oscillation frequencies differ from the analog relations for a simple sound field by the presence of a supplementary factor of the type

$$(1 + 2B^2b^{-1}\cos 2\theta_1 + B^4b^{-2})^{1/6}$$

while the distribution of the local heat-exchange coefficient over the surface of the cylinder remains symmetric. It can be shown that if $B^2b^{-1} > -2 \cos 2\theta_1$, the presence of a second oscillatory motion leads to enhancement of the heat-exchange process, while for $\theta_1 = 0$ or π maximum enhancement occurs for assigned values of b and B. When the oscillation planes are perpendicular to each other, and the amplitude-frequency characteristics satisfy the relation $B^2 = b$, the presence of a second oscillatory motion completely suppresses convective heat exchange.

Consider the case of identical frequencies (in this case it is primarily assumed that ($\varepsilon^2 \operatorname{Pr}$) > 1). Placing the coordinate system at the leading point of secondary flows and using relationship (31) of [2], Eq. (19) is written in the form

$$2hN_2\left[\sin\left(\sigma_4\pm\beta\right)\mp A\right]\frac{\partial t_{00}}{\partial\sigma_4} - h^2N_2\cos\left(\sigma_4\pm\beta\right)\frac{\partial t_{00}}{\partial h} = \frac{1}{2}\frac{\partial^2 t_{00}}{\partial h^2},\tag{23}$$

where $\sigma_4 = \pm (2\theta - \arcsin A - \arctan (D/C))$.

Introducing the variable

$$x_4 = \left(\frac{4N_2}{9}\right)^{1/3} h \frac{[\sin(\sigma_4 \pm \beta) \mp A]^{1/2}}{\left[\int_{0}^{\sigma_4} [\sin(\chi \pm \beta) \mp A]^{1/2} d\chi\right]^{1/3}},$$

Eq. (23) is reduced to an ordinary differential equation whose solution is described by relationship (20), where x_3 must be replaced by x_4 . The expression for the local heat-transfer coefficient then acquires the form

$$Nu_{a} = 0.6 \left(\frac{A_{1}^{2}}{\sqrt{v\omega_{1}}} \frac{a}{\vartheta}\right)^{1/3} N_{2} \frac{[\sin(\sigma_{4} \pm \beta) \mp A]^{1/2}}{\left[\int_{0}^{\sigma_{4}} [\sin(\chi \pm \beta) \mp A]^{1/2} d\chi\right]^{1/3}}.$$
(24)

It follows from (24) that, as in the case $(\epsilon^2 \text{ Pr}) \ll 1$, the distribution of the local heat-exchange coefficient over the cylinder surface is symmetric, which is due to the presence of a large-scale circulatory flow.

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